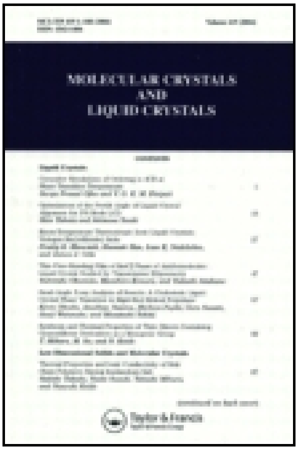

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Dynamics of Nematic Liquid Crystal under Oscillatory Flow: Influence of Surface Viscosity

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Dynamics of Nematic Liquid Crystal under Oscillatory Flow: Influence of Surface Viscosity

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We analyse the influence of a surface viscosity on the orientational dynamics of a nematic liquid crystal subjected to an oscillatory Couette flow. Approximate analytical solutions of nematohydrodynamic equations for small flow amplitudes are calculated and compared with the results of full numerical simulations. The range of flow frequencies where the surface viscosity has strong influence on the optical response is determined.

Keywords: nematic liquid crystal; oscillatory flow; surface viscosity

INTRODUCTION

In a nematic liquid crystal (NLC) complex flow behavior arises from the strong coupling between the velocity and orientational degrees of freedom (director $\hat{\mathbf{n}}$). Orientational dynamics of NLC under oscillatory flow has generally been studied in the case of strong surface anchoring (fixed orientation at the substrates). The influence of weak surface anchoring and surface viscosity on the dynamics of director reorientations under an applied electric field has been discussed in [1, 2]. The surface-dominated orientational dynamics in NLC embedded in a solid porous matrix has been studied by means of the dynamic scattering technique, from which the temperature behavior of surface viscosity has been deduced [3]. Recently, a flow-induced orientational transition in nematics at the substrate with weak planar anchoring was found and an estimate of surface viscosity has been given [4].

In this paper the orientational behavior of the NLC layer with weak surface anchoring subjected to the rectilinear oscillatory Couette flow is studied theoretically. We analyse the case of small flow amplitudes where the director motion is within the flow plane. The influence of anchoring strength and surface viscosity on the orientational dynamics are investigated and the appropriate experimental conditions to measure the surface friction are proposed.

BASIC EQUATIONS

The nematic layer of thickness d is confined between two identical substrates which provide weak surface anchoring at $z = \pm d/2$ in a Cartesian coordinate system. The oscillating flow is in the x direction and the director confined to $x - z$ plane. In this situation the director and the velocity are only functions of the distance z from the boundaries and time t

$$\begin{aligned} n_x = \cos\theta(z,t),\ n_y = 0,\ n_z = \sin\theta(z,t), \\ v_x = v_x(z,t),\ v_y = 0,\ v_z = 0, \end{aligned} \tag{1}$$

where $\theta(z,t)$ is the angle with respect to the x axis. With the dimensionless variables $\tilde{z} = z/d$, $\tilde{t} = t\omega$, $\tilde{v}_x = v_x/(\omega d)$ the equations governing the alignment and the flow can be rewritten as [5, 6]

$$\theta_{,t} - K(\theta) v_{x,z} = \varepsilon \left[P(\theta)\theta_{,zz} + \frac{1}{2}P'(\theta)\theta_{,z}^2 \right], \tag{2}$$

$$\delta v_{x,t} = \partial_z \Big\{ -(1-\lambda)K(\theta)\theta_{,t} + Q(\theta)v_{x,z} \Big\}, \tag{3}$$

where the tildes have been omitted and

$$K(\theta) = \frac{\lambda\cos^2\theta - \sin^2\theta}{1-\lambda}, \lambda = \frac{\alpha_3}{\alpha_2}, P(\theta) = \cos^2\theta + K\sin^2\theta, K = \frac{K_{33}}{K_{11}},$$

$$2(-\alpha_2)Q(\theta) = \alpha_4 + (\alpha_5 - \alpha_2)\sin^2\theta + (\alpha_3 + \alpha_6 + 2\alpha_1\sin^2\theta)\cos^2\theta,$$

$$\varepsilon = \frac{1}{\tau_d\omega}, \tau_d = \frac{\gamma_1 d^2}{K_{11}}, \gamma_1 = \alpha_3 - \alpha_2, \delta = \tau_v\omega, \tau_v = \frac{\rho d^2}{-\alpha_2}.$$

Here the α_i are the viscosity coefficients, K_{ii} are elastic constants, ρ is the density of NLC and the notation $f_{,i} \equiv \partial f/\partial i$, $f'(g) \equiv \partial f/\partial g$ has been used throughout. Boundary conditions for the velocity for the oscillatory Couette flow are

$$v_x(z=-1/2) = 0,\ v_x(z=+1/2) = a\cos(t), \tag{4}$$

where $a = A_x/d$ and A_x is the upper plate displacement amplitude.

The weak anchoring is described mathematically in terms of surface energy per unit area $F_s = (W/2)f_s(\theta - \theta_0)$, where W is the anchoring strength and function $f_s(\theta - \theta_0)$ has a minimum at $\theta = \theta_0$. A simple phenomenological expression for the surface energy was introduced by Rapini and Papoular where $f_s(\theta - \theta_0) = \sin^2(\theta - \theta_0)$ [7]. The boundary conditions for the director can be obtained from the surface torques balance equation [2, 8]

$$\mp P(\theta)\theta_{,z} + \frac{Wd}{2K_{11}}\frac{\partial f_s}{\partial \theta} + \frac{\omega d}{K_{11}}\eta\frac{\partial \theta}{\partial t} = 0, \tag{5}$$
$$- \text{ on } z = -1/2 \text{ and } + \text{ on } z = 1/2.$$

Here $\eta = \gamma_1 l_{\gamma_1}$ is so-called surface viscosity which characterises the dissipation at the substrate; γ_1 is the bulk orientational viscosity and l_{γ_1} is a characteristic interfacial length for surface viscosity.

In order to obtain an approximate analytical solution of Eqs.(2), (3) we consider the case of small flow amplitudes $a \ll 1$ which corresponds to the small distortions of the director profile

$$\theta = \theta_0 + \tilde{\theta}\ , \quad v_x = v_{x0} + U\ , \quad |\tilde{\theta}| \ll 1\ , \quad |U| \ll 1\ , \tag{6}$$

where $\theta_0 = const$, $v_{x0} = 0$ is the solution of (2), (3) in the absence of oscillatory flow ($a = 0$); the value θ_0 is defined by the minimum of surface energy given for small amplitude of the director oscillations as $f_s = \tilde{\theta}^2$. In the low-frequency range to be considered here one has $\delta \ll 1$ ($\rho \approx 10^3$ kg/m^3, $|\alpha_2| \approx 10^{-1}$ N·s/m^2 and $d \approx 10^{-5}$ m gives $\delta < 1$ for frequencies $f < 1$ kHz) and the inertia term [left-hand side of Eq.(3)] is neglected. Then one obtains from equations (2), (3)

$$\tilde{\theta}_{,t} - K_0 U_{,z} = \varepsilon P_0 \tilde{\theta}_{,zz},$$
$$(1 - \lambda)K_0\tilde{\theta}_{,tz} - Q_0 U_{,zz} = 0, \tag{7}$$

where $K_0 = K(\theta_0)$, $P_0 = P(\theta_0)$, $Q_0 = Q(\theta_0)$, with the boundary conditions

$$\tilde{\theta}_{,z} - E\tilde{\theta} - G\tilde{\theta}_{,t} = 0\ |_{z=-1/2}\ , \quad \tilde{\theta}_{,z} + E\tilde{\theta} + G\tilde{\theta}_{,t} = 0\ |_{z=1/2},$$
$$U(z = -1/2) = 0, \quad U(z = 1/2) = a\cos(t). \tag{8}$$

Here $E = Wd/(P_0K_{11})$, $G = \omega d\gamma_1 l_{\gamma_1}/(P_0K_{11})$. Since Eqs.(7) are linear, the periodic boundary conditions (8) (periodic forcing) will lead to time-periodic solutions for the director

$$\tilde{\theta}(z,t) = T_1(z)\cos(t) + T_2(z)\sin(t) \tag{9}$$

with

$$T_1(z) = aK_0 \frac{C_1F_1(z) - C_2F_2(z)}{C_1^2 + C_2^2}, \quad T_2(z) = -aK_0 \frac{C_1F_2(z) + C_2F_1(z)}{C_1^2 + C_2^2}$$

and for the velocity

$$\begin{aligned} U(z,t) &= U_1(z)\cos(t) + U_2(z)\sin(t), \\ U_1(z) &= a(\frac{1}{2} + z) + a\frac{(1-\lambda)K_0^2}{2Q_0k}\frac{C_1F_3(z) - C_2F_4(z)}{C_1^2 + C_2^2}, \\ U_2(z) &= -a\frac{(1-\lambda)K_0^2}{2Q_0k}\frac{C_1F_4(z) + C_2F_3(z)}{C_1^2 + C_2^2}, \end{aligned} \tag{10}$$

where the functions $F_i(z)$ and C_i are given in Appendix A.

RESULTS AND DISCUSSION

For weak homeotropic anchoring ($\theta_0 = \pi/2$, $W = 10^{-6}$ J/m^2) the profiles of the director deviation $T_1(z)$, $T_2(z)$ from the homeotropic position as well as the velocity components $U_1(z)$, $U_2(z)$ are presented in Fig. 1 for different values of the surface viscosity $\eta = \gamma_1 l_{\gamma_1}$. The material parameters of MBBA (see Appendix B) have been used throughout. The curves for strong homeotropic anchoring [fixed orientation at the substrates, $\tilde{\theta}(z = \pm 1/2) = 0$] as well as for zero surface viscosity ($l_{\gamma_1} = 0$) are also shown for comparison. For the velocity component U_1 we subtract the linear part $a(z + 1/2)$ which corresponds to the velocity profile in case of isotropic liquid. One can see that the value of surface viscosity strongly influence on the director and velocity distribution under the oscillatory Couette flow. Note, that raising of the surface viscosity has a similar effect as the increasing of the surface anchoring strength W.

The amplitude of the director oscillations is proportional to $K_0 = K(\theta_0)$. For MBBA one has $K(\pi/2)/K(0) \approx 10^2$ and the amplitude of the director oscillations for weak planar anchoring is much smaller than for the homeotropic orientation at the same flow amplitude a. Therefore, in case of planar orientation the influence of the surface viscosity on the orientational dynamic of NLC is much smaller than for weak homeotropic anchoring.

One of the widely used technique for studying the orientational behavior of NLC is the measurements of the transmitted through

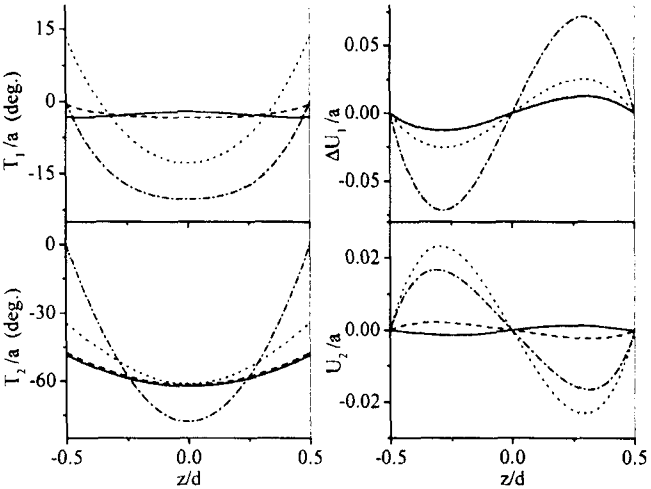

Figure 1: Director and velocity profiles for oscillatory Couette flow. $a = 0.2$, $f = 5$ Hz, $d = 10$ μm, $W = 10^{-6}$ J/m^2, l_{γ_1} [m]: 0 (—); 10^{-7} (- -); 10^{-6} ($\cdots$), strong anchoring (- · · -).

the NLC cell polarized light. In the geometry where the polars are crossed and the x axis is at 45° one has for the light intensity

$$I = I_0 \sin^2 \frac{\Psi}{2}\ , \quad \Psi = \frac{2\pi d}{\Lambda} \int_{-1/2}^{1/2} \left[\frac{n_o n_e}{\sqrt{n_o^2 \cos^2\theta + n_e^2 \sin^2\theta}} - n_o \right] dz\ , \quad (11)$$

where n_o, n_e are the ordinary and extraordinary refractive indices and Λ is the wavelength of light. Time-periodic director oscillations lead to a variation of the optical response $I = I(t)$. Using the director solution (9), the transmitted light intensity can easily be calculated. We found that depending on the value of surface viscosity, the maximum of the intensity changes and its position is shifted in time with respect to the moment of zero displacement of upper plate ($t = \pi/2$). In order to find the range of flow frequencies where the surface viscosity has a strong influence on the optical response, the dependence of the maximum of the transmitted light intensity I_{max} on the flow frequency was calculated for different values of surface viscosity (Fig. 2). We present the results for two typical values of anchoring strength W within the range of recently found experimental ones [3]. The optical response for the case of strong anchoring is

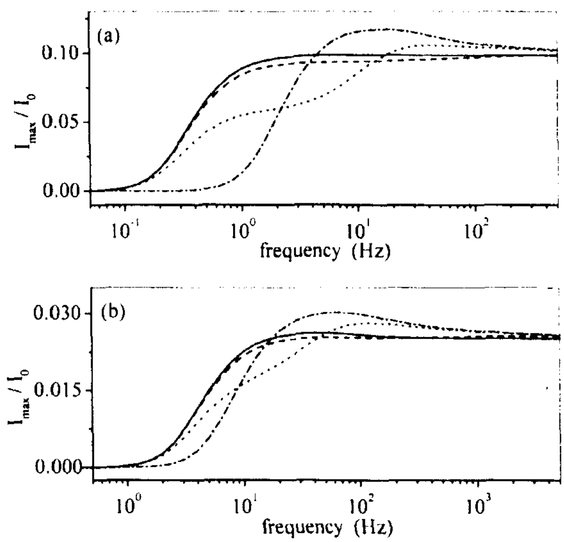

Figure 2: Maximum of the transmitted light intensity versus the frequency of oscillatory Couette flow at $a = 0.2$. (a) - $d = 10$ μm, $W = 10^{-6}$ J/m^2; (b) - $d = 5$ μm, $W = 10^{-5}$ J/m^2. l_{γ_1} [m]: 0 (—); 10^{-7} (- -); 10^{-6} (· · ·), strong anchoring (- · -).

shown for comparison. It is clearly seen that there exists some flow frequency range [$\sim 0.1 \div 10$ Hz in Fig. 2(a) and $\sim 1 \div 100$ Hz in Fig. 2(b)] where the optical response is strongly depending on the value of surface viscosity. Below this frequency range the transmitted intensity is small and the determination of the surface viscosity is complicated, whereas at higher frequencies the boundary layers of the order of $\sqrt{K_{11}/(2\omega\gamma_1)}$ at $z = \pm 1/2$ [6] become thinner and the influence of the surface viscosity on the director dynamics is not so apparent.

In order to verify the approximate solutions, direct numerical simulations of system (2), (3) with boundary conditions (4), (5) have been performed. It was found that for the frequency range $\omega \ll \tau_v^{-1}$ ($\delta \ll 1$) one can safely use the analytical small-amplitude solutions up to the flow amplitudes $a \approx 0.25$ for the homeotropic anchoring (the difference between analytical expressions and numerical solution does not exceed 0.5%).

In conclusion, the influence of the anchoring strength and surface viscosity on the director dynamics in the bulk of NLC and at the substrates is investigated for oscillatory Couette flow. Approximate analytical solutions for small-amplitude oscillatory flow in case of weak surface anchoring are obtained. The analysis shows that there exists a certain flow frequency range where the optical response strongly depends on the value of surface viscosity. This effect can be used for a more precise experimental determination of the surface viscosity and investigations of surface-dominated orientational dynamics in oscillatory flow of NLC.

Appendix A:

$$
\begin{aligned}
F_1(z) &= \cosh(kz)\cos(kz) - \cosh(k/2)\cos(k/2) + \\
&+ \frac{kE}{E^2+G^2}\Big\{\cosh(k/2)\sin(k/2) - \sinh(k/2)\cos(k/2)\Big\} - \\
&- \frac{kG}{E^2+G^2}\Big\{\cosh(k/2)\sin(k/2) + \sinh(k/2)\cos(k/2)\Big\},\\
F_2(z) &= \sinh(kz)\sin(kz) - \sinh(k/2)\sin(k/2) - \\
&- \frac{kE}{E^2+G^2}\Big\{\cosh(k/2)\sin(k/2) + \sinh(k/2)\cos(k/2)\Big\} - \\
&- \frac{kG}{E^2+G^2}\Big\{\cosh(k/2)\sin(k/2) - \sinh(k/2)\cos(k/2)\Big\},\\
F_3(z) &= \sinh(kz)\cos(kz) - \cosh(kz)\sin(kz) - \\
&-2z\Big(\sinh(k/2)\cos(k/2) - \cosh(k/2)\sin(k/2)\Big),\\
F_4(z) &= \sinh(kz)\cos(kz) + \cosh(kz)\sin(kz) - \\
&-2z\Big(\sinh(k/2)\cos(k/2) + \cosh(k/2)\sin(k/2)\Big),\\
k &= \sqrt{\frac{Q_0-(1-\lambda)K_0^2}{2\varepsilon Q_0 P_0}},\\
C_1 &= \sinh(k/2)\sin(k/2) - \\
&- \frac{(1-\lambda)K_0^2}{Q_0 k}\Big[\cosh(k/2)\sin(k/2) - \sinh(k/2)\cos(k/2)\Big] + \\
&+ \frac{kE}{E^2+G^2}\Big\{\cosh(k/2)\sin(k/2) + \sinh(k/2)\cos(k/2)\Big\} + \\
&+ \frac{kG}{E^2+G^2}\Big\{\cosh(k/2)\sin(k/2) - \sinh(k/2)\cos(k/2)\Big\},\\
C_2 &= \cosh(k/2)\cos(k/2) -
\end{aligned}
$$

$$-\frac{(1-\lambda)K_0^2}{Q_0 k}\Big[\cosh(k/2)\sin(k/2)+\sinh(k/2)\cos(k/2)\Big]-$$
$$-\frac{kE}{E^2+G^2}\Big\{\cosh(k/2)\sin(k/2)-\sinh(k/2)\cos(k/2)\Big\}+$$
$$+\frac{kG}{E^2+G^2}\Big\{\cosh(k/2)\sin(k/2)+\sinh(k/2)\cos(k/2)\Big\}\,.$$

Appendix B:

The numerical computations are carried out for the following MBBA material parameters at 25 °C [9, 10]: viscosity coefficients in units of 10^{-3} N·s/m^2 : $\alpha_1 = -18.1$, $\alpha_2 = -110.4$, $\alpha_3 = -1.1$, $\alpha_4 = 82.6$, $\alpha_5 = 77.9$, $\alpha_6 = -33.6$; elasticity coefficients in units of 10^{-12} N: $K_{11} = 6.66$, $K_{22} = 4.2$, $K_{33} = 8.61$; mass density $\rho = 10^3$ kg/m^3 and refractive indices for wavelength of light $\Lambda = 670$ nm: $n_o = 1.542$, $n_e = 1.7435$ [11].

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